How Time Dilation Generates Earth's Gravity

Thomas R. Mathews, PE
Mathews Engineering
9/7/2021

Abstract

Albert Einstein said that the acceleration due to gravity results from the energy that a falling object gains as it moves into an area of dilated time. This is often summed up as: "Space-time tells matter how to move and matter tells space-time how to curve". This paper will show a simple calculation that reveals that, because matter slows the local passage of time this causes the acceleration of nearby objects.

For the case where $r \geq r_e$ The time dilation caused by a massive object (such as the earth) can be expressed as:

$$\frac{t_0}{t_f} = \sqrt{1 - \frac{2GM}{rc^2}} \quad \text{(Equation 1)}$$

Where:

- G: The gravitational constant ($6.674 \times 10^{-11}\ m^3 kg^{-1} s^{-2}$)
- M: The mass of the earth (5.972×10^{24} Kg)
- r: The distance from the center of the earth
- c: The speed of light (299.792458×10^6 m/sec)
- r_e: Distance of the earth's surface from its center (6.378×10^6 m)
- t_0: The proper time
- t_f: The time as measured by a distant observer

At the surface of the earth, the time dilation due to earth's gravity is:

$$\frac{t_0}{t_f} = \sqrt{1 - \frac{2GM}{r_e c^2}} = 0.999999999305 = -695\, ppt$$

-695 parts per trillion (ppt) is a miniscule amount of time dilation. Time dilation at the earth's surface is also affected by the fact that the earth's surface is in motion. For an object orbiting just above the earth's surface the effect due to orbital velocity adds an additional -347 ppt. For the purpose of showing that the acceleration due to gravity is explained solely by time-dilation, this paper will not consider the dilation due to velocity and, for simplicity, the earth is treated as a non-rotating object.

The time dilation due to earth's gravity is shown in Figure 1 for both the case $r < r_e$ and $r \geq r_e$. The equation for $r < r_e$ is slightly different than Equation 1 (because this would be within the earth's mass). This version of the equation is not shown here but is reflected in Figure 1.

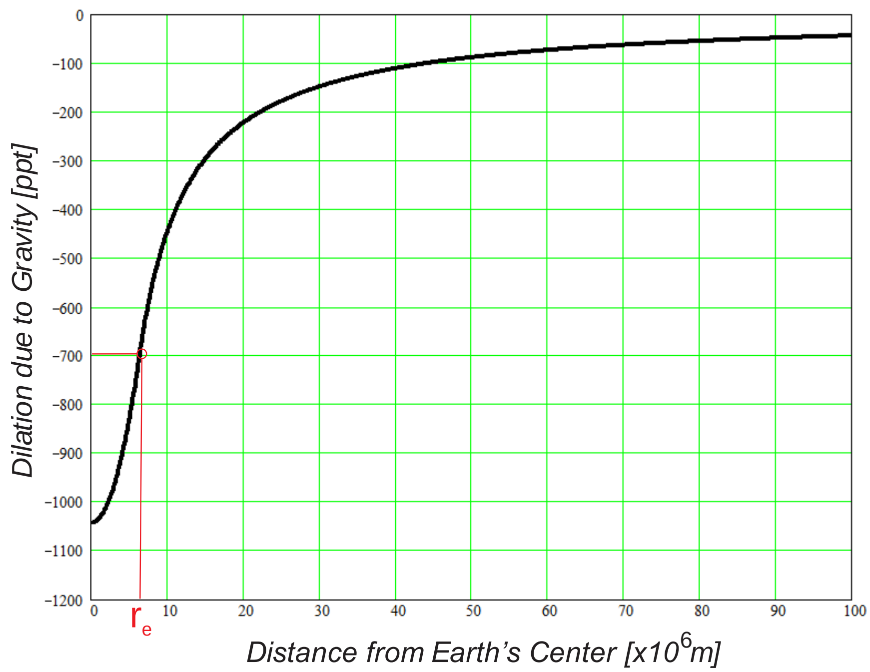

Figure 1. Time Dilation Due to Earth's Gravity

The acceleration of gravity at the earth's surface is widely known to be 9.8 m/sec^2. Everyone who lives on the earth knows this force well, and it is indeed a substantial force. So how can a miniscule time dilation of -695 parts per trillion (ppt) be responsible for such a large acceleration force?

In four-space physics, the objects that we consider stationary are stationary only in space, and move through time at the speed of light. In fact, modern 4-space physics demands that the speed of all objects is always the speed of light (c)—it is only the object's direction that changes. That is, an object is never truly stationary, it is only that its speed vector is directed totally in the time direction and we perceive this as "stationary". This also explains why matter contains so much energy. The famous mass energy equivalence equation "$E=mc^2$" arises because, in nuclear reactions, some of this "light speed" motion (through time) is redirected into space as energetic particle's ejected from an atom's nucleus.

To show how the acceleration of earth's gravity is due to these tiny time dilations, consider an object with mass (m) falling toward the earth from a very large distance r:

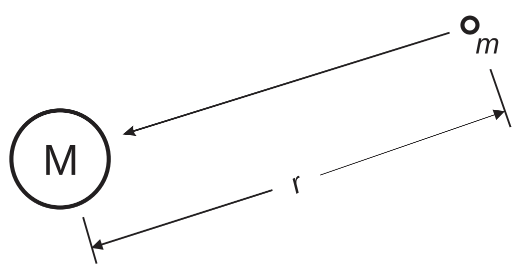

Figure 2. Object Falling to Earth from a Large Distance

The potential energy of this object at any distance r is:

$$PE = U(r) = -\frac{GMm}{r}$$

The speed of this object, originating from an infinite distance, and falling directly towards the earth can be calculated using:

$$KE + PE = 0$$

$$KE = \frac{1}{2}mv^2$$

$$\frac{1}{2}mv^2 + \left(-\frac{GMm}{r}\right) = 0$$

Solve for v(r):

$$v(r) = \sqrt{\frac{2GM}{r}}$$ *(Equation 2)*

Now imagine that, rather than a small object falling in from infinity, that space itself is "spilling" into the earth at the velocity given by equation 2.

Consider now that the time dilation due to equation 1 also creates an implied velocity of the space that is spilling into the earth v(r) that can be calculated using the Lorentz relationship. This relationship is shown in Figure 3 where velocities have been scaled to the speed of light 'c'. Notice that, since the speed of light is a very large number, very little dilation is required for v(r) to be a significant value. For example, at the earth's surface $v(r_e)$ is 11,180 m/sec (about 25,000 mph).

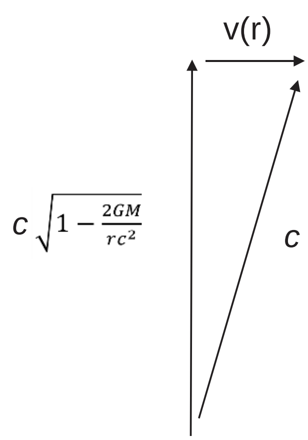

Figure 3. Implied Velocity v(r) Due to Lorentz Time Dilation

v(r) can be calculated from Figure 3 using the Pythagorean Theorem:

$$\left(c\sqrt{1-\frac{2GM}{rc^2}}\right)^2 + v(r)^2 = c^2$$

$$c^2 - \frac{2GM}{r} + v(r)^2 = c^2$$

$$v(r)^2 = \frac{2GM}{r}$$

$$v(r) = \sqrt{\frac{2GM}{r}}$$ *(Equation 3)*

At this point, we might notice that Equation 3 is identical to Equation 2 but let's proceed and calculate the acceleration that Equation 3 implies:

$$a(r) = \frac{d}{dt}v(r) = \frac{d}{dt}\left(\frac{2GM}{r}\right)^{1/2}$$

Applying the chain rule for derivatives:

$$a(r) = \frac{1}{2}\left(\frac{2GM}{r}\right)^{-\frac{1}{2}}(-2GMr^{-2})\frac{dr}{dt}$$

Note that dr/dt is simply the velocity given by Equation 3. Substitute this for dr/dt to get:

$$a(r) = \frac{1}{2}\left(\frac{2GM}{r}\right)^{-\frac{1}{2}}(-2GMr^{-2})\left(\frac{2GM}{r}\right)^{\frac{1}{2}}$$

Cancelling the $(2GM/r)^{\pm 1/2}$ terms leads to:

$$a(r) = -\frac{GM}{r^2}$$

Substitute $r = r_e$ to get the astounding result:

$$a(r_e) = -9.803\,\frac{m}{s^2}$$

This is the acceleration that we feel every day and, as shown here, is a direct result of tiny time dilations.

Conclusion

In the vicinity of large amounts of mass, the progression of time is slowed. This dilation results in the familiar gravitational acceleration of nearby objects. But why does matter slow time? and how does this effect extend far out into space? To date there are no good answers to this question. Stephen Wolfram proposes that space-time is a network constructed on hypergraph theory. It seems locality would not allow any such network to be synchronously clocked. One can imagine that an asynchronously clocked hypergraph, in the vicinity of large amounts of matter, would require additional time to update events and could result in the time dilation that we observe.

References

Misner, C. W. (1973). *Gravitation.* W.H. Freeman and Company.

Wikipedia. (2021). Retrieved from Gravitational constant: https://en.wikipedia.org/wiki/Gravitational_constant

Wikipedia. (2021). Retrieved from Gravitational energy: https://en.wikipedia.org/wiki/Gravitational_energy

Wikipedia. (2021). Retrieved from Gravitatioinal time dilation: https://en.wikipedia.org/wiki/Gravitational_time_dilation

Wolfram, S. (2020). *A Project to Find the Fundamental Theory of Physics.* Wolfram Media, Inc.